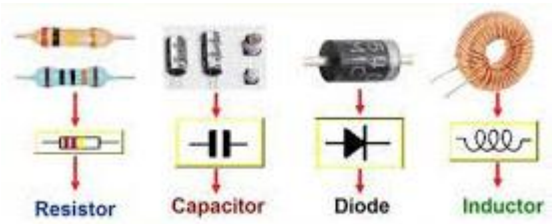
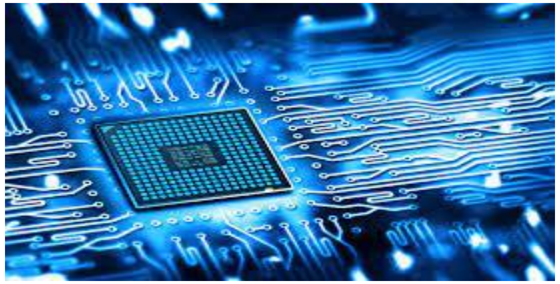
Electronics 1

BSC 113

Summer 2021-2022

Lecture 11





NATURAL AND STEP RESPONSE FOR RL, RC AND RLC CIRCUIT

INSTRUCTOR

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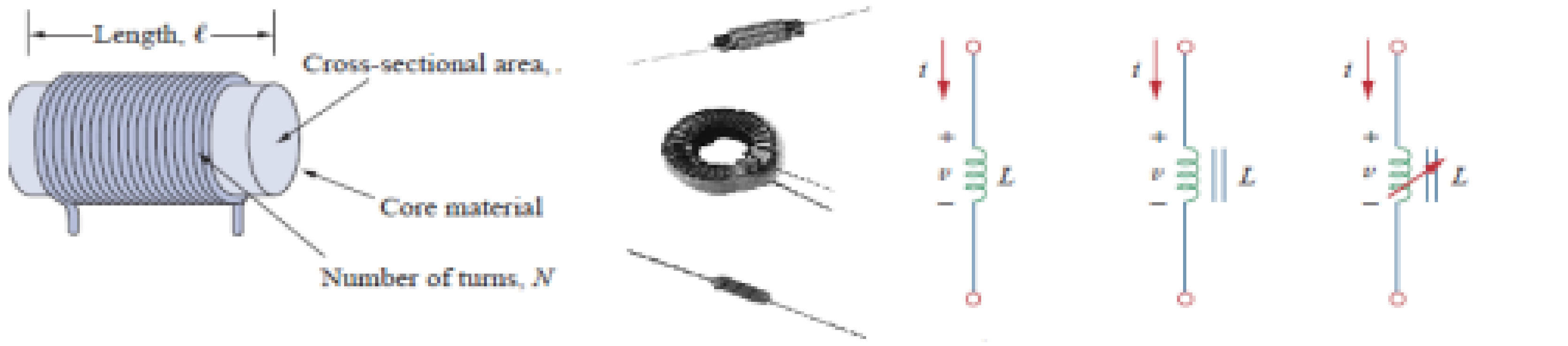
Contents

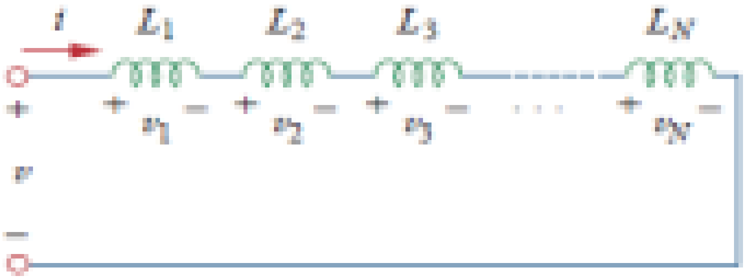
Inductors

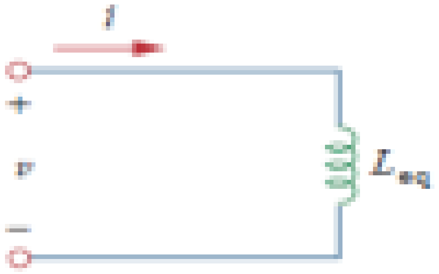
- An inductor is a passive element designed to store energy in its magnetic field. Inductors find numerous applications in electronic and power systems.
- They are used in power supplies, transformers, radios, TVs, radars, and electric motors.

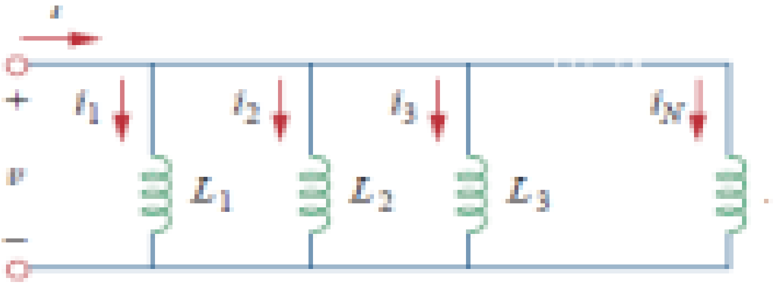
- Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire



Inductors







$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt}$$

$$= (L_1 + L_2 + L_3 + \cdots + L_N)\frac{di}{dt}$$

$$= \left(\sum_{k=1}^{N} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

$$i = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0) + \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt + i_N(t_0)$$

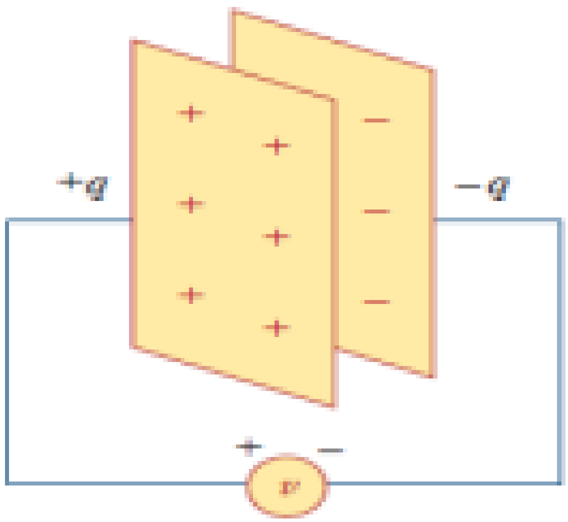
$$= \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i(t_0)$$

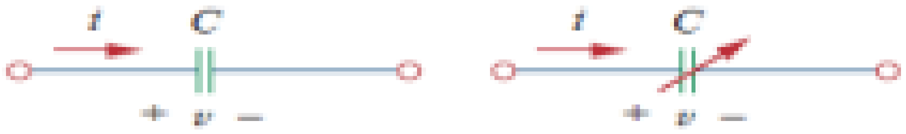
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

Capacitors

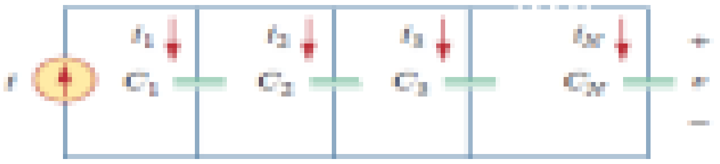
- A capacitor is a passive element designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components. Capacitors are used extensively in electronics, communications, computers, and power systems.

$$1F = 1C/1V$$





Capacitors

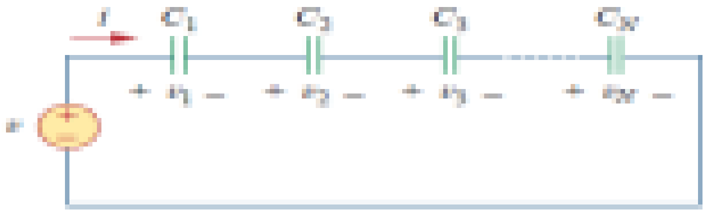


$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_k\right)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$



$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(t)\,dt + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(t)\,dt + v_2(t_0)$$

$$+ \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(t)\,dt + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(t)\,dt + v_1(t_0) + v_2(t_0)$$

$$+ \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(t)\,dt + v(t_0)$$

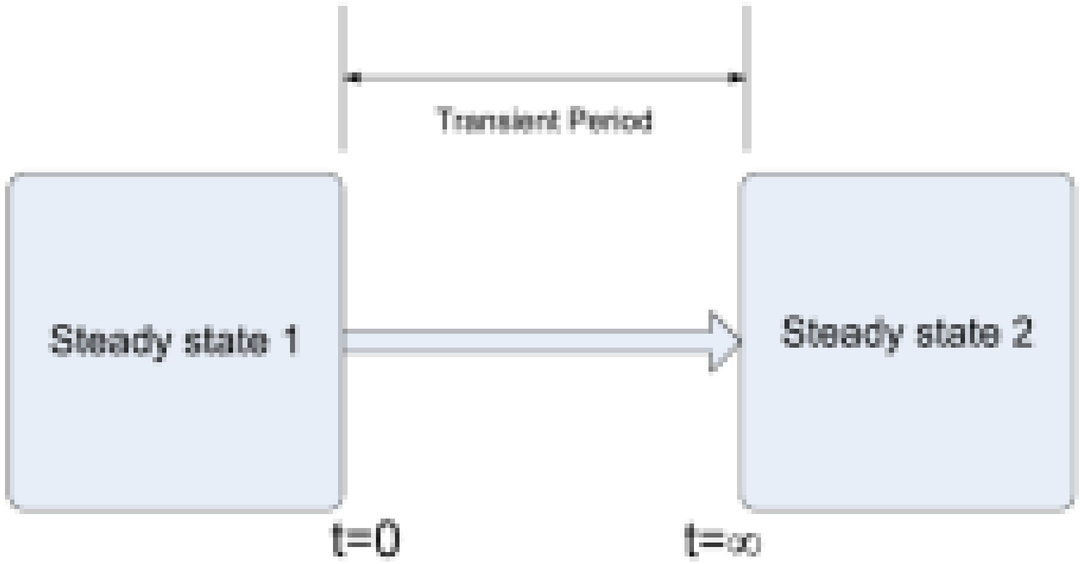
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

Important characteristics of basic elements

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit

First order transient circuit

- In this chapter, we shall examine two types of simple circuits: a circuit comprising a resistor and capacitor and a circuit comprising a resistor and an inductor.
- These are called RC and RL circuits, respectively.
- A first-order circuit is characterized by a first-order differential equation.



1- Source free R-C circuit

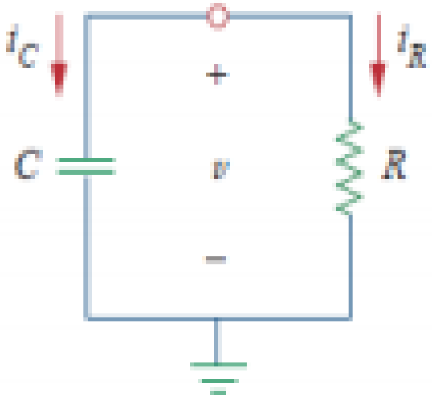
- A source-free RC circuit occurs when its dc source is suddenly disconnected.

- The energy already stored in the capacitor is released to the resistors (sometimes called free response).

- This shows that the voltage response of the RC circuit is an exponential decay of the initial voltage.

- Since the response is due to the initial energy stored and the physical characteristics of the circuit and not due to some external voltage or current source, it is called the natural response of the circuit.

1- Source free R-C circuit



$$v(0) = V_0$$

$$w(0) = \frac{1}{2}CV_0^2$$

$$i_C + i_R = 0$$

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} = 0$$

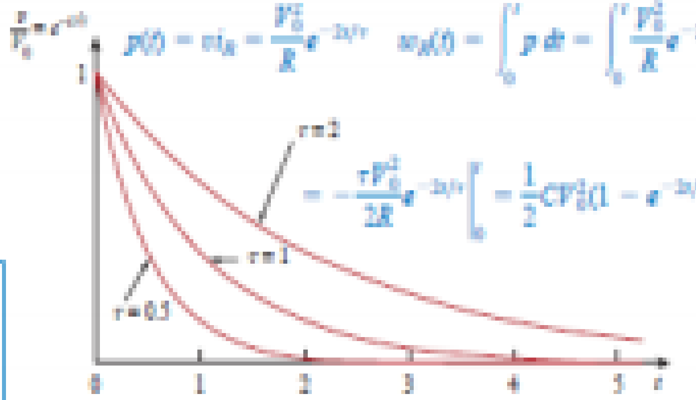
$$\frac{dv}{v} = -\frac{1}{RC}dt$$

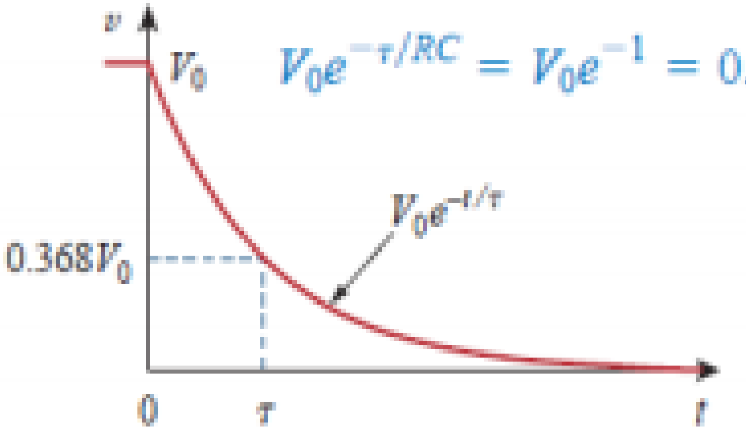
$$\ln\frac{v}{A} = -\frac{t}{RC}$$

$$v(t) = Ae^{-t/RC}$$

$$v(t) = V_0 e^{-t/RC}$$

$$p(t) = v i_R = \frac{V_0^2}{R}e^{-2t/\tau} \qquad w_R(t) = \int_0^t p\,dt = \int_0^t \frac{V_0^2}{R}e^{-2t/\tau}dt = -\frac{\tau V_0^2}{2R}e^{-2t/\tau}\Big|_0^t = \frac{1}{2}CV_0^2(1 - e^{-2t/\tau})$$





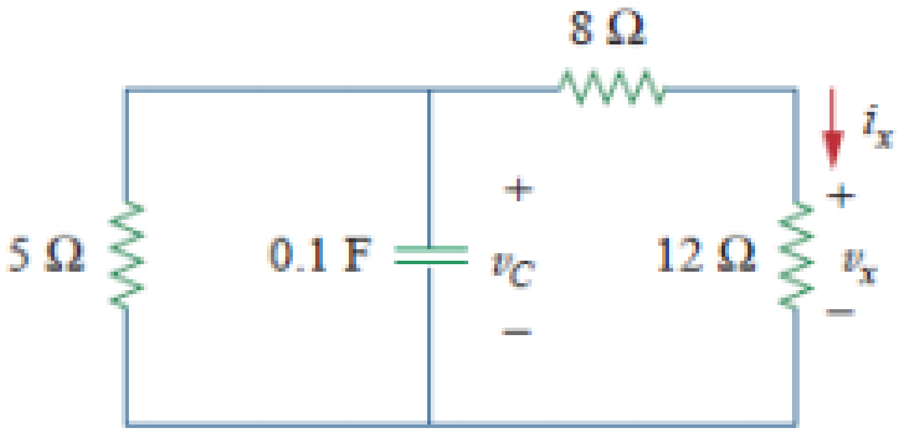
$$V_0 e^{-\tau/RC} = V_0 e^{-1} = 0.368V_0$$

$$\tau = RC$$

$$v(t) = V_0 e^{-t/\tau}$$

Example

- let $V_c(0)=15$, find V_c, V_x and i_x for $t>0$



$$R_{eq} = \frac{20 \times 5}{20 + 5} = 4\ \Omega$$

$$\tau = R_{eq}C = 4(0.1) = 0.4\ \text{s}$$

$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\ \text{V}, \qquad v_C = v = 15e^{-2.5t}\ \text{V}$$

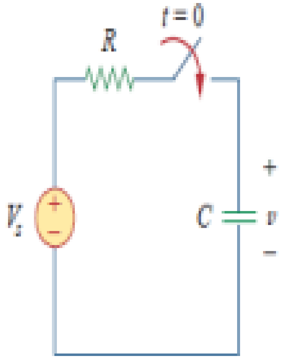
$$v_x = \frac{12}{12+8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\ \text{V}$$

$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\ \text{A}$$

2- Forced R-C circuit

- The steady-state response is the behavior of the circuit a long time after an external excitation is applied.

- The forced R-C circuit is shown in figure. with v (∞) is the final value of capacitor voltage.



$$v(0^-) = v(0^+) = V_0$$

$$C\frac{dv}{dt} + \frac{v - V_s u(t)}{R} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC}u(t)$$

$$\frac{dv}{dt} = -\frac{v - V_s}{RC}$$

$$\frac{dv}{v - V_s} = -\frac{dt}{RC}$$

$$\ln(v - V_s)\Big|_{V_0}^{v(t)} = -\frac{t}{RC}\Big|_0^t$$

$$\ln(v(t) - V_s) - \ln(V_0 - V_s) = -\frac{t}{RC} + 0$$

$$\ln\frac{v - V_s}{V_0 - V_s} = -\frac{t}{RC}$$

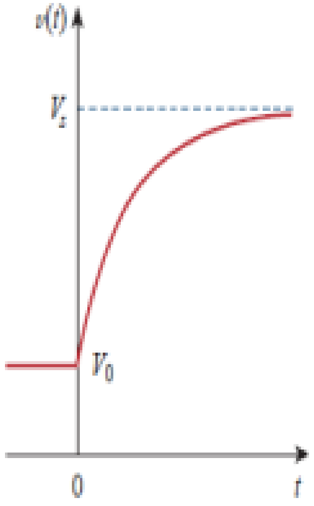
$$\frac{v - V_s}{V_0 - V_s} = e^{-t/\tau}, \qquad \tau = RC$$

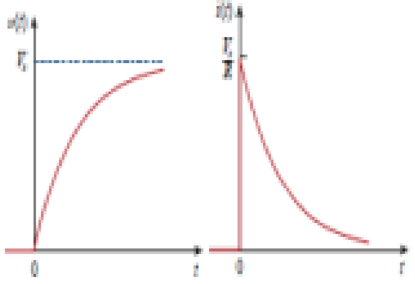
$$v - V_s = (V_0 - V_s)e^{-t/\tau}$$

$$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}, \qquad t > 0$$

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau}, & t > 0 \end{cases}$$

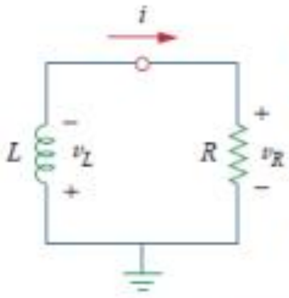
$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$





3- Source free R-L circuit

- Consider the series connection of a resistor and an inductor, as shown in Fig.
- Our goal is to determine the circuit response.



$$i(0) = I_0$$

$$w(0) = \frac{1}{2} L I_0^2$$

$$v_L + v_R = 0$$

$$L\frac{di}{dt} + Ri = 0$$

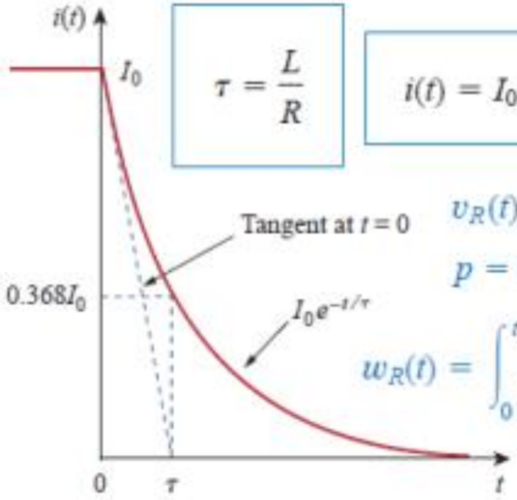
$$\frac{di}{dt} + \frac{R}{L} i = 0$$

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L} dt$$

$$\ln i \Big|_{I_0}^{i(t)} = -\frac{Rt}{L}\Big|_0^t$$

$$\ln \frac{i(t)}{I_0} = -\frac{Rt}{L}$$

$$i(t) = I_0 e^{-Rt/L}$$



$$\tau = \frac{L}{R}$$

$$i(t) = I_0 e^{-t/\tau}$$

$$v_R(t) = iR = I_0 R e^{-t/\tau}$$

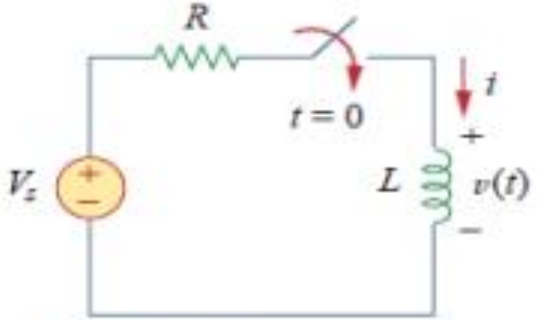
$$p = v_R i = I_0^2 R e^{-2t/\tau}$$

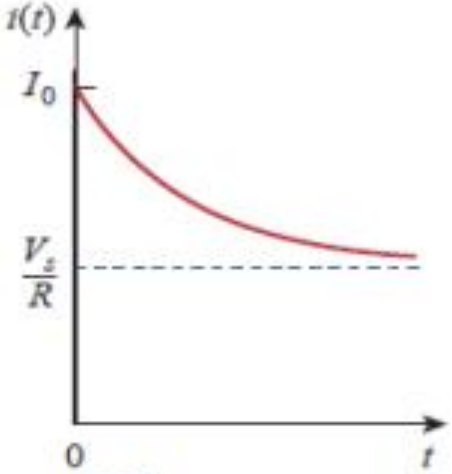
$$w_R(t) = \int_0^t p\,dt = \int_0^t I_0^2 R e^{-2t/\tau} dt = -\frac{1}{2}\tau I_0^2 R e^{-2t/\tau}\Big|_0^t$$

$$w_R(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau})$$

4 Forced R-L circuit

- The steady-state response is the behavior of the circuit a long time after an external excitation is applied. The forced R-L circuit is shown in figure with i (∞) is the final value of inductor current.





$$i = i_t + i_{ss}$$

$$i_t = Ae^{-t/\tau}, \qquad \tau = \frac{L}{R}$$

$$i_{ss} = \frac{V_s}{R}$$

$$i = Ae^{-t/\tau} + \frac{V_s}{R}$$

$$i(0^+) = i(0^-) = I_0$$

$$I_0 = A + \frac{V_s}{R}$$

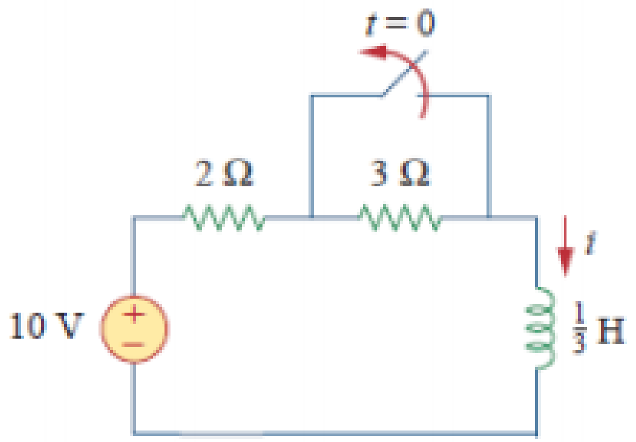
$$A = I_0 - \frac{V_s}{R}$$

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

Example

- Find i(t) in the circuit in the following Fig. for t >0. Assume that the switch has been closed for a long time.



Answer: When t < 0 the 3-ohm resistor is short-circuited, and the inductor acts like a short circuit. The current through the inductor is

$$i(0) = \frac{10}{2} = 5A$$

When t > 0 the switch is open.

$$i(\infty) = \frac{10}{2+3} = 2A$$

The Thevenin resistance across the inductor terminals is

$$R_{Th} = 2 + 3 = 5$$

Thus,

$$i(t) = 2 + 3e^{-15t} A$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

Second order transient circuit

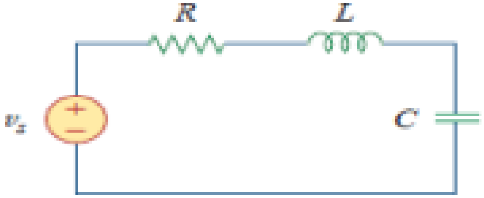
- In this chapter we will consider circuits containing two storage elements. These are known as second-order circuits because their responses are described by differential equations that contain second derivatives.

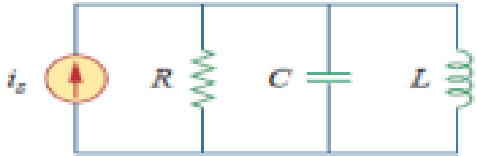
- Typical examples of second-order circuits are RLC circuits, in which the three kinds of passive elements are present.

- Examples of such circuits are shown in figure. A second-order circuit is characterized by a second-order differential equation.

- It consists of resistors and the equivalent of two energy storage elements.

Second order transient circuit





$$i = Ae^{st}$$

$$v(0) = \frac{1}{C}\int_{-\infty}^{0} i\,dt = V_0$$

$$As^2e^{st} + \frac{AR}{L}se^{st} + \frac{A}{LC}e^{st} = 0$$

$$i(0) = I_0$$

$$Ae^{st}\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) = 0$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt = 0$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$Ri(0) + L\frac{di(0)}{dt} + V_0 = 0$$

$$s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$$

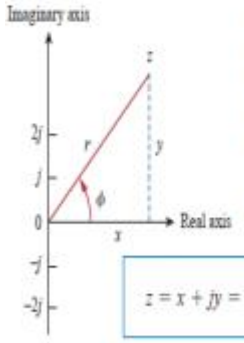
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

SINUSOIDAL STEADY STATE ANALYSIS

1- Phasors and sin wave

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.



$z = x + jy$ Rectangular form

$z = r\angle\phi$ Polar form

$z = re^{j\phi}$ Exponential form

$r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}$

$x = r\cos\phi, \quad y = r\sin\phi$

$$z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)$$

Subtraction: $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$

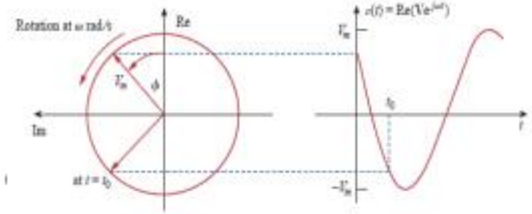
Multiplication: $z_1 z_2 = r_1 r_2 \angle\phi_1 + \phi_2$

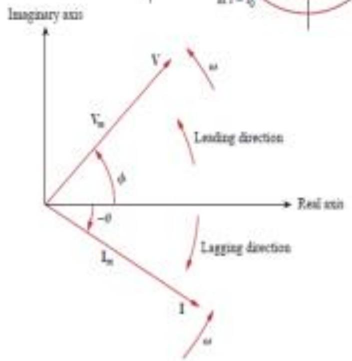
Division: $\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\phi_1 - \phi_2$

Reciprocal: $\frac{1}{z} = \frac{1}{r}\angle -\phi$

Square Root: $\sqrt{z} = \sqrt{r}\angle\phi/2$

Complex Conjugate:
$z^* = x - jy = r\angle -\phi = re^{-j\phi}$



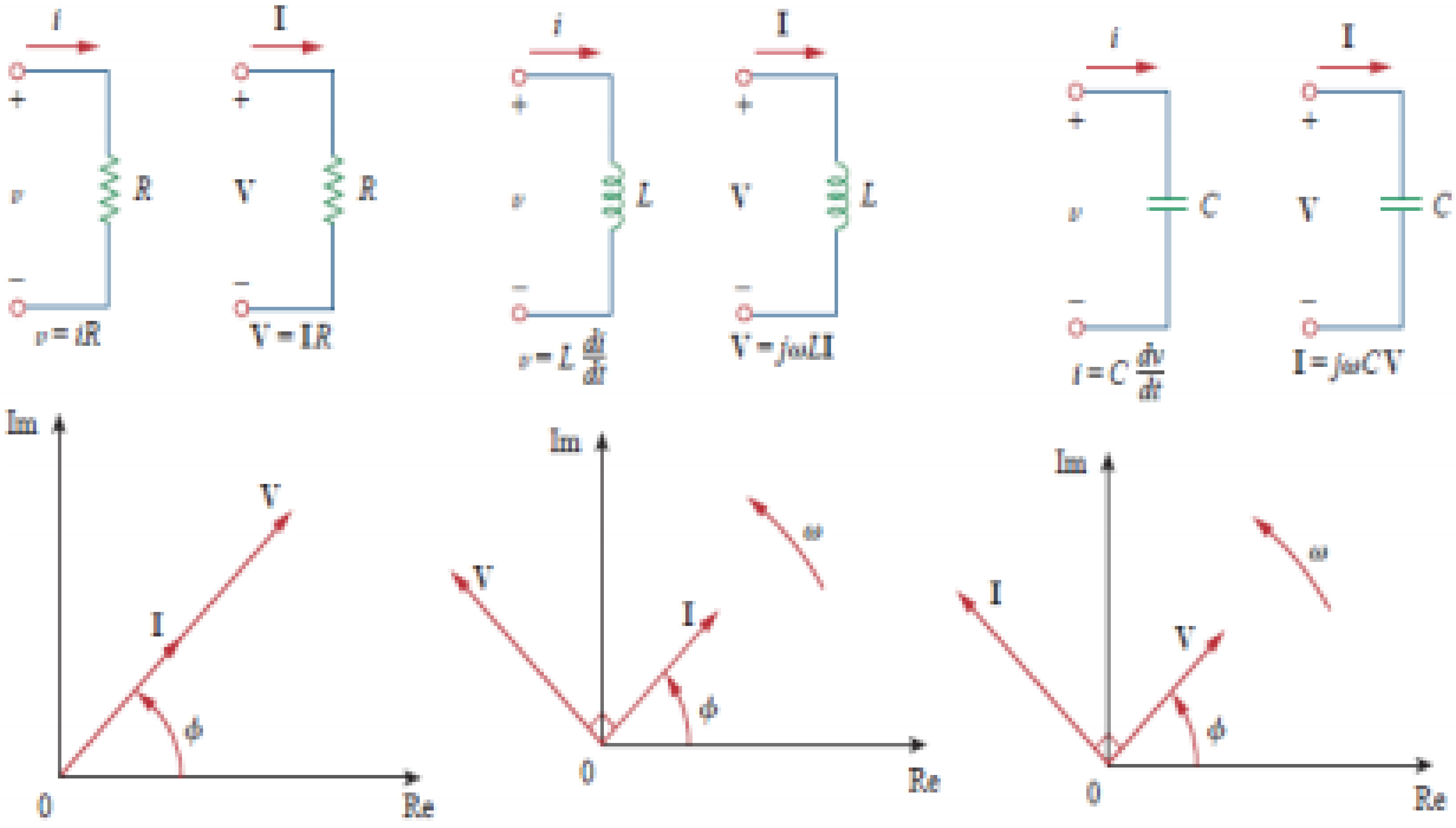


Sinusoid-phasor transformation.

Time domain representation	Phasor domain representation
$V_m\cos(\omega t + \phi)$	$V_m\angle\phi$
$V_m\sin(\omega t + \phi)$	$V_m\angle\phi - 90^\circ$
$I_m\cos(\omega t + \theta)$	$I_m\angle\theta$
$I_m\sin(\omega t + \theta)$	$I_m\angle\theta - 90^\circ$

2- Electrical circuit analysis

- Phasor relationships of circuit elements



2- Electrical circuit analysis

➢ Summary of voltage-current relationships

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

3- Average and RMS

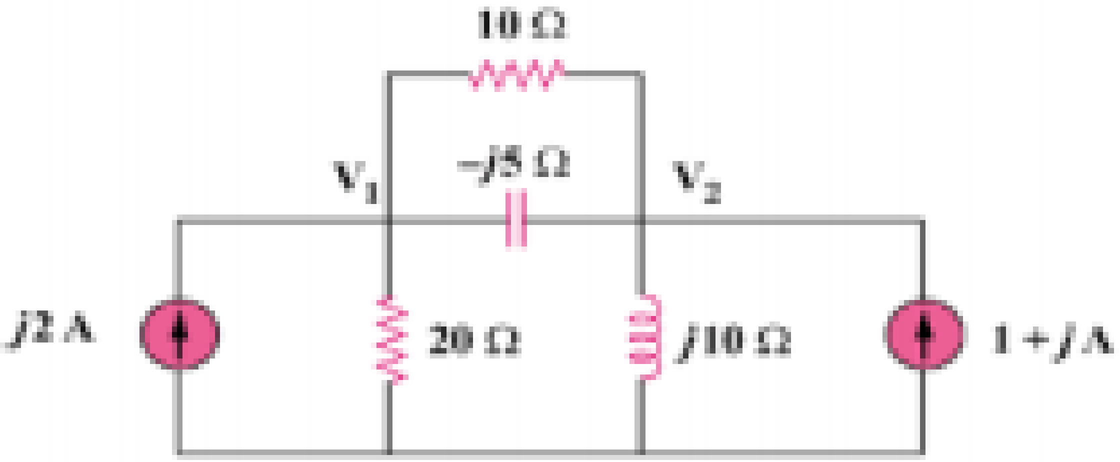
- The two important parameter in electronics calculations is average and root mean square value which are discussed here.

$$v_{avg} = \frac{1}{T}\int_0^T v(t)dt$$

- Given a periodic function , its rms value (or the effective value) is given by

$$v_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

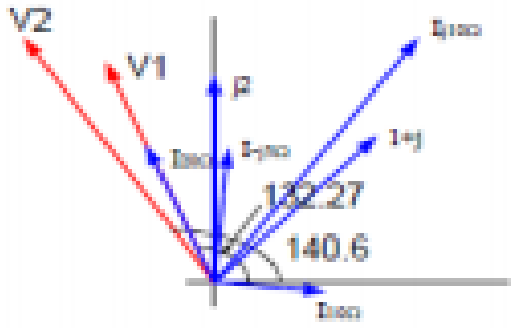
Example



At node 1,

$$j2 = \frac{V_1}{20} + \frac{V_1 - V_2}{10} + \frac{V_1 - V_2}{-j5}$$

$$j40 = (3 + j4)V_1 - (2 + j4)V_2$$



$V_1 = \underline{22.87\angle 132.27^\circ \text{ V}}$

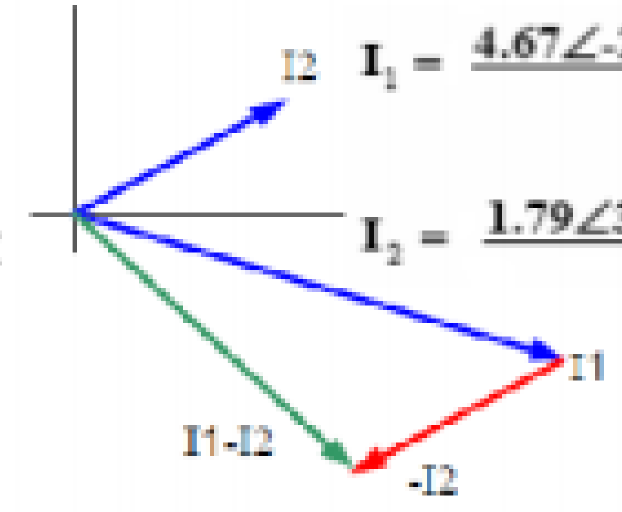
$V_2 = \underline{27.87\angle 140.6^\circ \text{ V}}$

At node 2,

$$\frac{V_1 - V_2}{10} + \frac{V_1 - V_2}{-j5} + 1 + j = \frac{V_2}{j10}$$

$$10(1 + j) = -(1 + j2)V_1 + (1 + j)V_2$$

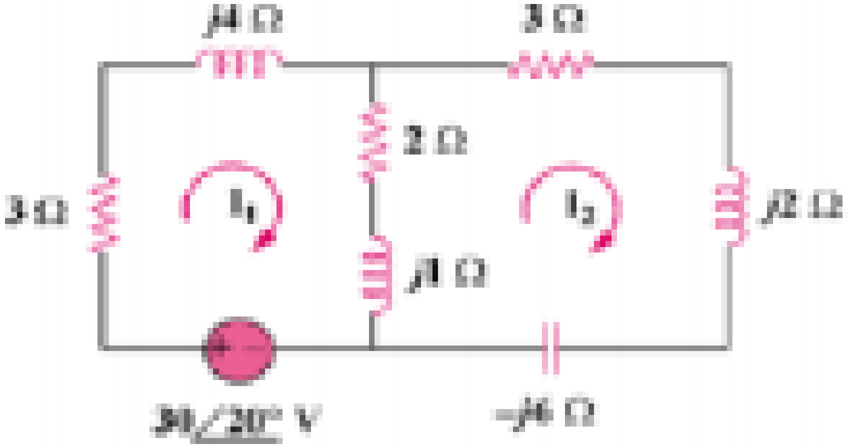
Example



(4)

$I_1 = \underline{4.67\angle{-20.17^\circ}\ A}$

$I_2 = \underline{1.79\angle{37.35^\circ}\ A}$



For mesh 1,

$$(5 + j5)\mathbf{I}_1 - (2 + j)\mathbf{I}_2 - 30\angle 20^\circ = 0$$

$$30\angle 20^\circ = (5 + j5)\mathbf{I}_1 - (2 + j)\mathbf{I}_2 \quad (1)$$

For mesh 2,

$$(5 + j3 - j6)\mathbf{I}_2 - (2 + j)\mathbf{I}_1 = 0$$

$$0 = -(2 + j)\mathbf{I}_1 + (5 - j3)\mathbf{I}_2 \quad (2)$$


Thank you